Public Finance Exercises
Second part's exercises

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1 Taxation: Progressivity and Redistribution

1. Consider an economic system with four individuals exhibiting the following distribution of pre-tax incomes:

$$Y^* = \{2; 4; 8; 16\}$$

(a) Assuming that the tax system is characterized by a deduction $d = 2$ for all individuals and a constant legal tax rate $t = 50\%$, complete the following table and state whether the conditions to guarantee *progressivity* are satisfied:

Gross income (Y)	**Net income (Y - d)**	**Tax Liability (T)**	**Effective t**	**Post-tax income**
2				
4				
8				
16				

(b) Draw the Lorenz curve for gross income L_Y, the Lorenz curve for the tax liability L_T, and the Lorenz curve for post-tax income L_{Y-T}: based on the *Jakobsson-Fellman-Kakwani* theorem, what can you conclude about the progressivity of this tax system?

(c) Assume now that the government decides to obtain the same post-tax incomes using a *tax rebate* f rather than a deduction d, without any changes in the legal tax rate t. Compute this tax rebate f.

(d) We are now interested in comparing Y^* with the distribution $Y^{**} = \{6; 7; 8; 9\}$. After drawing the Lorenz curves associated with Y^* and Y^{**}, identify the *socially preferable* distribution and explain why.

2 Social Choice

1. Consider a society composed of 100 individuals, with preferences:

- Left: $a \succ b \succ c \rightarrow n_l = 25$
- Center-left: $b \succ a \succ c \rightarrow n_{cl} = 20$
- Center-right: $b \succ c \succ a \rightarrow n_{cr} = 15$
- Right: $c \succ b \succ a \rightarrow n_r = 40$

(a) Calculate the choice that results as the Condorcet winner.

(b) Calculate the choice preferred by society according to the simple majority rule, if all individuals vote sincerely.

(c) Calculate the choice preferred by society according to the runoff rule between the two most voted options, if all individuals vote sincerely.

(d) Calculate the choice preferred by society according to a Borda count method where each individual assigns 2 points to their preferred alternative, 1 point to the second choice, and 0 points to the least preferred alternative, if all individuals vote sincerely.

(e) Describe an example (for each of the three previous voting systems) of how the groups supporting the losing alternatives have the opportunity to strategically manipulate the vote to their advantage.

3 Electoral Competition

1. In a society composed of 3 individuals, there are two parties, L (*Left*) and R (*Right*), competing under the simple majority rule. Each voter $i \in \{1, 2, 3\}$ has a utility function: $U_i = c_i + ln(G)$, where c_i represents private consumption and G is a public good. The individual budget constraint for individual i is $c_i = (1-t)w_i$, where w_i represents the income of individual i and t represents a proportional tax rate. Assume the income of individual 1 is $w_1 = 1$, the income of individual 2 is $w_2 = 4$, and the income of individual 3 is $w_3 = 9$. The government's budget constraint is $G = t\sum_{i=1}^{3} w_i$. The parties propose t and G before the elections, and are bound to implement the proposed policy if they win the elections.

   (a) What level of t and G will the parties propose, if they are *office-motivated*, before the elections? Solve the model.

   (b) What level of t and G will the parties propose, if they are *policy-motivated*, before the elections? Specifically, assume the utility function of party L is $U_L = -(t-1)^2$, while the utility function of party R is $U_R = -(t^2)$. Express the objective function and constraints of each of the two parties. It is not necessary to solve the model.

   (c) What would be the efficient level of t and G according to a utilitarian social welfare function? Solve the model.

   (d) Assume now that the probability that individual 1 goes to the polls is 1; the probability that individual 2 goes to the polls is 1/2, and the probability that individual 3 goes to the polls is 1/3. What level of t and G will the parties propose, if they are *office-motivated*, before the elections? Solve the model.

4 Legislative Bargaining

1. Suppose that in a parliamentary democracy, three parties are present in parliament. We will call them c (*Centre*), l (*Left*), and r (*Right*). None of them obtained a majority in the elections, but a coalition formed by any two parties (of the three) is necessary and sufficient to form a government. The parties can negotiate the division of a public budget $R = 100$ among the working-class (represented by party l), the middle class (represented by party c), and the upper class (represented by party r). The utility function of each party p is equal to $U_p = r_p$, where r_p represents the amount of public budget resources allocated to the social group represented by party p. Suppose further that events occur in the following order: one party is randomly selected (with probability $p_p = \frac{1}{3}$ equal for each) as the first proposer. If a coalition of two parties is formed, the parties derive utility from the allocation of funds. Otherwise, a new party is randomly chosen as proposer (with probability $p_p = \frac{1}{3}$ equal for all parties). If a coalition of two parties is formed, the parties derive utility from the allocation of funds. Otherwise, all parties receive a utility of 0.

   (a) How would resources be distributed if the game reached the final period?

   (b) Calculate the unique subgame perfect equilibrium of this dynamic game.

(c) Now assume that if an agreement is not reached even in the second period, all parties receive a utility of $U_p = 25$. Calculate the unique subgame perfect equilibrium of this dynamic game.