

# Public Finance

## First part's exercises

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### 1 Tragedy of the Commons

1. Consider two individuals  $i = \{1, 2\}$  who can purchase rifles ( $S_i$ ) for hunting. Each rifle costs  $c$ . The number of prey that can be hunted by each rifle is an increasing and concave function of the total number of rifles purchased by the two individuals  $F(S)$ , where  $S = S_1 + S_2$ . For the purpose of the exercise, we assume that  $F(S) = 2\sqrt{S}$ .
  - (a) Calculate the optimal number of rifles purchased privately by individual 1 and individual 2. Calculate the total number of rifles purchased by the society if individuals make their decisions privately.
  - (b) Calculate the socially efficient number of rifles.

### 2 Public Goods

1. Two individuals  $i = 1, 2$  have the same income  $Y_i = 100$ , which can be used in part to purchase shirts and in part to contribute to the hiring of police officers. For the purpose of the exercise, let's assume that the utility function of individual  $i$  is  $U_i = 3 \log(s_i) + 6 \log(P)$ , where  $s_i$  is the number of shirts purchased by individual  $i$  and  $P = P_1 + P_2$  is the total number of police officers hired by a society whose members are individual 1 and individual 2.
  - (a) Calculate the optimal private choice of  $s_1, s_2, P_1, P_2$ .
  - (b) Calculate the socially efficient choice according to Samuelson's rule. *You are free to decide whether to apply Samuelson's formula or to solve the society's optimal problem.*
  - (c) Draw the two individuals' reaction curves and identify graphically the Nash equilibrium and the socially efficient choice.

### 3 Principles of Sacrifice

1. A society is formed by individuals belonging to the upper class and individuals belonging to the working class. All individuals have the same utility function  $U(Y_i) = \sqrt{Y_i}$  but different income levels. In particular, the income of individuals from the upper class is  $Y_u = 400$  while the income of individuals from the working class is  $Y_w = 100$ . Now, suppose the government decides to introduce a proportional tax rate  $t_u = 10\%$  on the income of individuals from the upper class.
  - What tax rate must be applied to individuals from the working class to equalize absolute sacrifice?
  - What tax rate must be applied to individuals from the working class to equalize proportional sacrifice?
  - What is the minimum tax rate that can be applied to individuals from the upper class to equalize marginal sacrifice if the government decides to exempt individuals from the working class from the tax?