Public Finance

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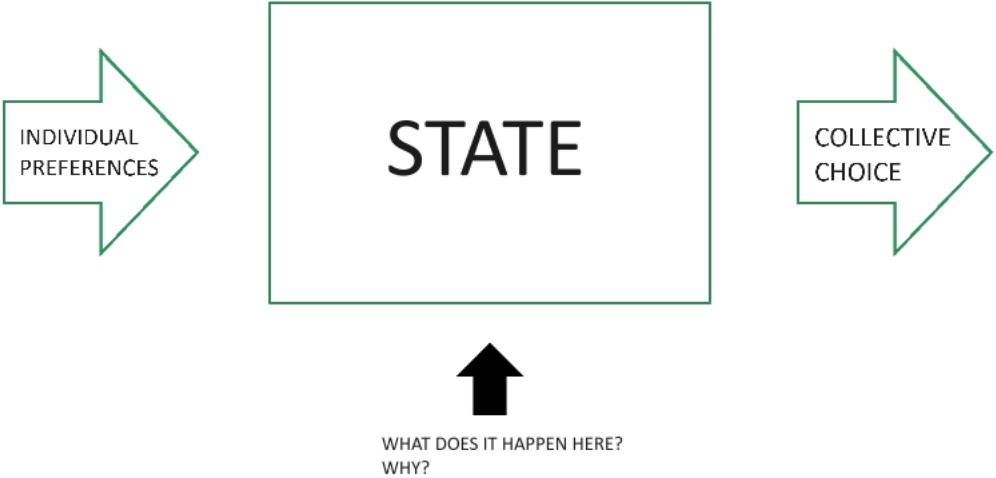
Theory of collective choices 1

Social Choice Theory

- Governments have various tools to intervene in the presence of market failures or undesirable distribution
- For example, in the case of externalities:
  1. Assignment of property rights
  2. Pigovian tax
  3. Pigovian transfer
  4. Quantity regulation
  5. Emission tax

  ...
- Which tool **will be chosen** by the government? It depends...

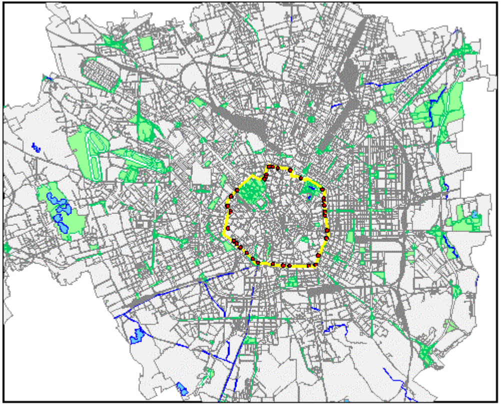
Social Choice Theory



Example: Milan's Area C

- All vehicles (excluding electric and hybrid ones) entering the center of Milan must pay a toll of 7.5 euros
- The toll applies each time a vehicle crosses the boundaries of the city center
- This is a clear example of intervention aimed at reducing **negative externalities**
  - Driving a car brings private benefits but contributes to pollution
  - Driving a car causes traffic problems for all other users (other drivers, public transport users, cyclists, pedestrians)
  - Buying an electric car incurs a higher private cost but generates fewer externalities
- This policy has something in common with a Pigovian tax and something in common with an emission tax
  - All accesses are subject to tax (not just those beyond an efficient level)
  - Incentives to reduce emissions are also present
    1. No payment for electric car drivers even if they contribute to traffic
    2. The municipality uses revenues to fund public transport, bike sharing, etc.

Example: Milan's Area C



- Area C covers approximately 4.5% of Milan's territory; 77K residents

Example: Milan's Area C

- **Normative question:** Is Area C the optimal way to address externalities from car use in a small, densely populated city center?
- We don't know. However, we do know that it was introduced in 2012 and is still active.
- Why? Area C seems to be a politically feasible way to address the issue. (**positive question**)
  - Approved by a referendum
  - Survived a legal process following a challenge to its establishment
  - Maintained in effect by various administrations of different political colors
  - Replaced a proper emission tax in effect until 2011 for which there was not enough political consensus

Example: Milan's Area C

- Many economic agents have interests in the matter
  - Car owners
  - Municipal politicians
  - Merchants and service providers
  - Historic center residents
  - Public transport users and pedestrians
  - ...
- To design **effective policies**, it is not possible to stop at the search for **efficiency**. It is also necessary to consider which policies are **feasible** in such complex contexts.

Social Choice Theory

- In the introductory part of the course, we saw how each society can order every possible state of the economy by defining a social welfare function
- For every state A and B, it is said that state A is strictly preferred by society over state B if $W(A) > W(B)$
- Different societies can express different preference orders, and the preferences of the same society can change over time
- Defining a SWF that holds concretely for a society is difficult!

Social Choice Theory

- Consider K individuals who must take a collective decision **regarding** a set of N alternative options (*states of the world*)
- Each individual has rational preferences that order the states of the world
- Each individual is different from others in many dimensions, including preferences and constraints
- To order states of the world according to a criterion, society must agree on a system that can aggregate individual preferences (i.e., on a social welfare function)

The Arrow Axioms

- Arrow (1950) proposed a series of axioms that mechanisms for aggregating individual preferences should satisfy
- Arrow's Axioms:

1. **Unlimited preference domain**
2. **Weak Pareto principle**
3. **Non-dictatorship**
4. **Independence of irrelevant alternatives (IIA)**

Arrow's Axioms: Unlimited Preference Domain

Definition

A social choice satisfies the axiom of the unlimited preference domain if any rational individual preference (i.e., complete, symmetric, and transitive) is allowed and considered to calculate the social order. The choice function produced by aggregation must, in turn, satisfy the rationality criterion.

- **Completeness**: Ensures that collective choice is **effective**, i.e., always able to determine which alternative should be chosen: if $\forall x \in X$ and $\forall y \in Y$ $x \succeq y$, or $y \succeq x$ then the social choice must result in a decision
- **Symmetry**: Ensures that preferences are independent of the order in which they are presented. If $x \succeq y$, it should also be the case that $y \preceq x$.
- **Transitivity**: If $x \succeq y$ and $y \succeq z$, then x must be (weakly) preferred to z.

Arrow's Axioms: Weak Pareto Principle

Definition

A social choice function satisfies the weak Pareto principle if, for every pair of alternatives x and y, the social choice prefers x to y if all individuals prefer x to y.

- Note the analogy with the concept of Pareto efficiency: social choice between x and y satisfies the weak Pareto principle if society prefers the efficient choice x to the inefficient y.
- According to the weak Pareto principle, the social choice cannot be dominated in a Pareto sense by any other (among the possible choices)
- The Pareto principle does not limit how society should decide between x and y when at least one individual prefers y to x.

Arrow's Axioms: Non-Dictatorship

Definition

A social choice function satisfies the non-dictatorship axiom if the social choice does not coincide with the preferences of a single individual regardless of the preferences of others.

- The collective choice criterion must avoid allowing an individual to make decisions based on their own preferences regardless of others' preferences.
- Note: the axiom does not exclude the possibility that there is a decisive individual who overturns the outcome of a decision (e.g., a decisive voter)
- Also note how the axiom does not exclude the possibility of having two dictators. Indeed, how could these two aggregate their preferences?

Arrow's Axioms: Independence of Irrelevant Alternatives

Definition

A social choice function satisfies the Independence of Irrelevant Alternatives (IIA) axiom if considering or not considering alternative z does not change the choice order between x and y.

- The axiom tends to exclude the possibility that the social choice can be manipulated (strategic voting, agenda power)
- Example: Society prefers an apple to a peach
  1. Society prefers a **apple** to an orange to a peach ✓
  2. Society prefers an orange to an apple to a peach ✓
  3. Society prefers a **apple** to a peach to an orange ✓
  4. Society prefers an orange to a peach to an apple ×
  5. Society prefers a peach to an orange to a apple ×
  6. Society prefers a peach to a apple to an orange ×

Does Majority Voting Satisfy All Arrow's Axioms?

Definition

Majority Voting: Mechanism used to aggregate individual votes whereby alternatives are subject to a vote by all individuals and the option receiving the majority of votes is chosen

- **Majority voting** is certainly the most commonly used mechanism to aggregate individual preferences, so it is useful to understand its advantages and limitations compared to Arrow's benchmark

Majority Voting

- Majority voting easily applies to binary choices. However, if we consider Arrow's axioms of the domain of unrestricted preferences and independence of irrelevant alternatives, choices are usually not restricted to two.
- There are various majority voting methods useful for ranking more than two options:
  1. Simple majority rule
     - ★ Among the various alternatives, society chooses the one preferred by the relative majority of individuals
     - ★ It is not necessary that more than half of individuals support the chosen alternative
  2. Runoff rule
     - ★ Among the various alternatives, if none is chosen by more than half of individuals, some alternatives are eliminated, and voters vote again until more than half of them support a single choice

Majority Voting

3. Binary choice system
   - Individuals choose between each possible pair of alternatives. The winner of all comparisons is chosen by society.
4. Sequential binary voting
   - Two alternatives are selected to be voted on in the first round. The most voted one survives and is voted against another. So on until all possible alternatives have been voted on
5. Point voting (Borda method)
   - Individuals rank their preferences and assign a decreasing number of points to each of them. The alternative receiving the maximum score is chosen by society.

Strategic Voting

Type	Preferences	Size
Left	$a \succ b \succ c$	10
Center-left	$b \succ a \succ c$	8
Center-right	$b \succ c \succ a$	6
Right	$c \succ b \succ a$	20

- **Simple Majority Rule:** if no one "cheats," c wins. But what happens if the left votes for b?
- **Runoff between the top two:** b wins unless the right shifts 5 votes to a in the first round
- The outcome depends on the ability to coordinate and behave strategically
  - Majority voting when more than two choices are available incentivizes strategic voting (i.e., IIA is violated)

Majority Voting and Condorcet Winner

Definition

In majority voting, an alternative is defined as the **Condorcet winner** if it is the chosen alternative compared to any other in a binary competition.

- x is the Condorcet winner if $x \succ y$ and $x \succ z$.
- Note that the Condorcet winner might not be chosen by society if other majority voting methods are adopted.
  - x might be the second-best alternative for those preferring y and the second-best alternative for those preferring z, but it is not necessarily the preferred choice by a relative majority of voters
  - x would win any runoff but does not necessarily qualify

Majority Voting and Condorcet Winner

Example

- Suppose there are 3 voters in town: 1, 2, 3
- Voters have different preferences on education spending levels (A, B, C)
  - Voter 1: $A \succ B \succ C$
  - Voter 2: $B \succ C \succ A$
  - Voter 3: $B \succ A \succ C$
- The town can proceed by sequentially voting on each alternative to reach a social choice
  - Vote between A and B: **B wins**
  - Vote between A and C: **A wins**
  - Vote between B and C: **B wins**
- **B is the winner** (Condorcet winner) as it has beaten both A and C and thus **majority voting has aggregated individual preferences** to produce a social choice.

Majority Voting and Condorcet Winner

- Which Arrow axioms have we satisfied?
  - Unlimited preference domain?
  - Weak Pareto principle ✓
  - Non-dictatorship ✓
  - Independence of irrelevant alternatives ✓

Majority Voting and Condorcet Winner

Example

- Let's assume there are 3 voters in the city: 1, 2, 3
- Voters have different preferences on education spending levels (A, B, C)
  - Voter 1: $A \succ B \succ C$
  - Voter 2: $C \succ B \succ A$
  - Voter 3: $B \succ C \succ A$
- The city can proceed by sequentially voting on each alternative to reach a social choice
  - Vote between A and B: **B wins**
  - Vote between A and C: **C wins**
  - Vote between B and C: **B wins**
- B has beaten both A and C, and therefore **B is the winner** (Condorcet winner)
- **Majority voting has aggregated individual preferences** to produce a social choice.

Majority Voting and Condorcet Winner

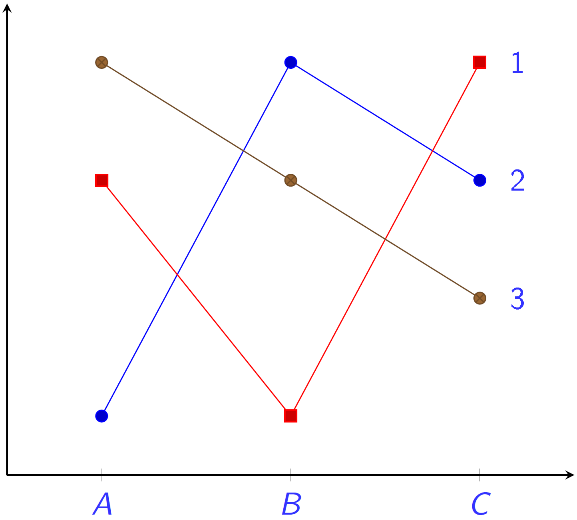
- Which Arrow axioms have we satisfied?
  - Unlimited preference domain?
  - Weak Pareto principle ✓(not violated)
  - Non-dictatorship ✓
  - Independence of irrelevant alternatives ✓

Majority Voting and No Condorcet Winner

Example

- Suppose there are 3 voters in the city: 1, 2, 3
- Voters have different preferences on education spending levels (A, B, C)
  - Voter 1: $C \succ A \succ B$
  - Voter 2: $B \succ C \succ A$
  - Voter 3: $A \succ B \succ C$
- The city can proceed by voting sequentially on each alternative to reach a social choice
  - Vote between A and B: **A wins**
  - Vote between A and C: **C wins**
  - Vote between B and C: **B wins**
- No alternative can beat every other in pairwise competitions. Condorcet cycle
- **Majority voting fails to aggregate individual preferences.**

Majority Voting and No Condorcet Winner



- Condorcet cycles are also known as the *Condorcet voting paradox*

Majority Voting and No Condorcet Winner

- Which Arrow axioms have we satisfied?
  - Unlimited preference domain ×
  - Weak Pareto principle ✓
  - Non-dictatorship ✓
  - Independence of irrelevant alternatives ✓
- **Majority voting needs the individual preference domain to be limited for the other three axioms to be satisfied simultaneously**

Sequential Majority Voting

- **Sequential voting:** A vote is taken between A and B. Then, the winner is voted against C
  - **C wins** $\rightarrow$ unlimited individual preference domain
  - Choice needs to have > 1 individuals supporting A over B and > 1 individuals supporting C over A $\rightarrow$ non-dictatorship
  - Weak Pareto principle is not violated
  - IIA is violated (the order between A and B depends on whether C is considered or not)
- Agenda-setting power is important
  - Voting on amendments before voting on a reform $\neq$ voting on a reform and then discussing amendments

Sequential majority voting does not guarantee IIA to be satisfied when the other three axioms are

Point Voting

- A single round of voting where each voter expresses a preference over all alternatives, assigning a decreasing number of points. The alternative receiving the highest number of points is the winner.
- **Advantages**:
  - The unlimited preference domain axiom is satisfied
- **Disadvantages**:
  - **IIA is not satisfied**, and there are tangible incentives for strategic voting
- Let's analyze a specific case of point voting: the ***Borda method***
  - Each voter ranks alternatives by assigning a score to each of them
  - Let N be the number of alternatives: the voter assigns N points to the favorite, $N-1$ to the second favorite, and so on until assigning 1 point to the least favorite alternative

Borda Method

Example

	w	x	y	z
Preferences of A	**4**	**3**	**2**	**1**
Preferences of B	**2**	**1**	**4**	**3**
Preferences of C	**1**	**4**	**3**	**2**
Total	**7**	**8**	**9**	**6**
Social choice (Borda)	3°	2°	1°	4°

- The total number of points received by each alternative is used to form the social order.
- All alternatives are comparable, the order is transitive, and there is no need to restrict the preference domain.

Borda Method

Example

- We can easily see how the relative order between x and y is not independent of the presence/absence of w and z)
  - Individual 1: $w \succ x \succ y \succ z$
  - Individual 2: $y \succ z \succ w \succ x$
  - Individual 3: $x \succ y \succ z \succ w$
- Individuals 1 and 3 prefer x to y. Nevertheless, y is preferred by society as x receives only 1 point from individual 2.

Borda Method

Example

	w	*x*	*y*
Preferences of A	3	2	1
Preferences of B	2	1	3
Preferences of C	1	3	2
Total	6	6	6
Social choice (Borda)	?	?	?

- **Let's exclude the option z from the alternatives**
- All remaining alternatives receive 7 points: **Social choice is not independent of the irrelevant alternative z**. Adding z, society makes a decision; otherwise, society fails to decide.

Arrow's Impossibility Theorem

Theorem

Any social choice mechanism that satisfies the axioms of unlimited domain, IIA, and weak Pareto principle must be dictatorial.

- Arrow's impossibility theorem doesn't exclude the possibility of obtaining a functional social choice mechanism. It only excludes the possibility of obtaining a social choice mechanism that works **in every circumstance**.
- **Does this mean dictatorship is a good system? Absolutely not!** But we must understand that any non-dictatorial system must necessarily limit the validity of one of the other three axioms.

Arrow's Impossibility Theorem

- Arrow's impossibility theorem doesn't preclude the possibility of achieving a functional social choice mechanism. It only precludes the possibility of achieving a social choice mechanism that functions **in every circumstance**.
- Allowing limited violations of either the unlimited preference domain or IIA may help identify functional choice mechanisms (excluding extreme cases).
  - For example, majority voting can work if we exclude the case of Condorcet cycles.
- Is it preferable to limit the domain of admissible preferences or IIA?

Restricting the Domain of Preferences

- Violating IIA is more problematic than it seems
  - Preferences would need to be considered cardinal and comparable across individuals
  - For example: point voting. Why should we decide that the distance between the favorite and the second option is equal for all individuals?
- Restricting the domain of preferences proves to be less problematic than it seems
  - Not restricting individual preferences is important
  - However, the Arrow impossibility result is due to the fact that even the most incredible preferences would be admissible to the computation.

Restricting the Domain of Preferences

- Consider the previous example and assume that A, B, and C are levels of investment in public education
  - A=high; B=medium; C=low
- Consider all possible ways of ordering the three alternatives allowed if the domain of preferences is unlimited
  1. $A \succ B \succ C$
  2. $A \succ C \succ B$
  3. $B \succ C \succ A$
  4. $B \succ A \succ C$
  5. $C \succ A \succ B$
  6. $C \succ B \succ A$
- (2) and (5) are not compatible with how we usually think of utility functions (unimodal)

Median Voter Theorem

- In the example, we are simplifying Arrow's criteria in two ways:
  - A, B, C can be ordered in the same dimension (monetary)
  - Excluding (2) and (5) implies that we restrict the domain of preferences to **unimodal preferences**
- In the presence of these two restrictions there is always a **Condorcet winner**

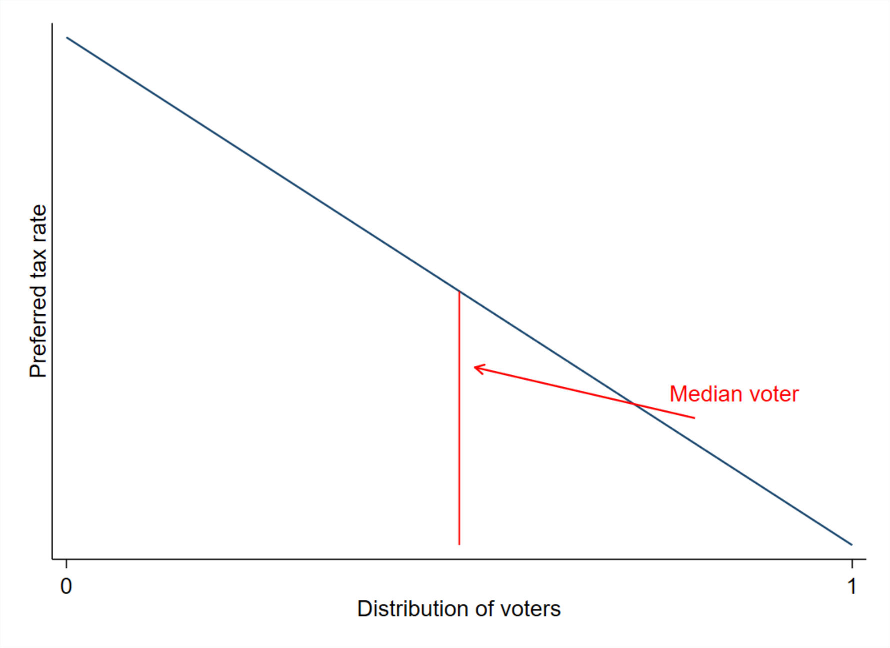
Theorem

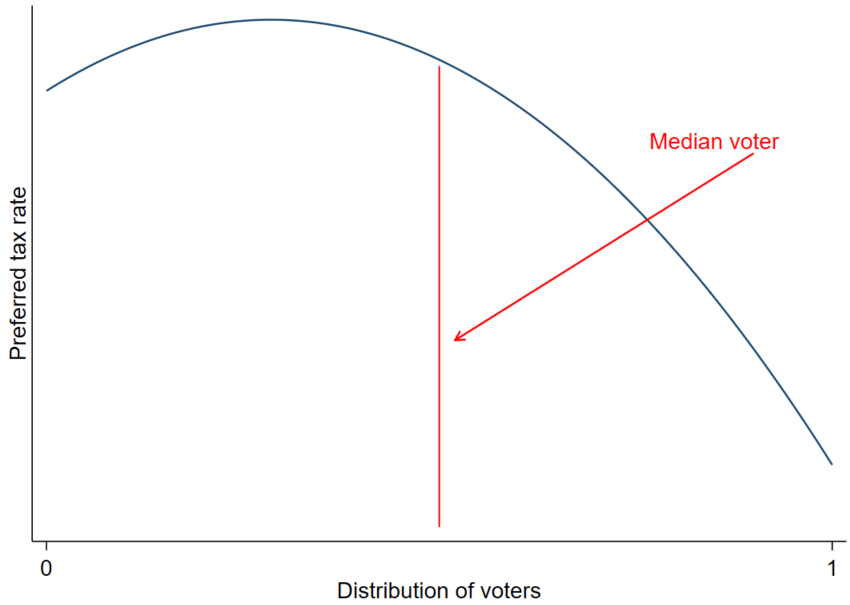
*If voters' preferences are **unimodal** in a one-dimensional space:*

- *The **social choice function** resulting from majority voting is **transitive**. In other words, there is always a Condorcet winner.*
- ***The Condorcet winner** is the alternative preferred by the median voter.*
- *The median voter is the individual for whom the alternative preferred by (K-1)/2 voters is not greater than the alternative preferred by the median voter, and the alternative preferred by (K-1)/2 voters is not less than the alternative preferred by the median voter.*

Median Voter Theorem

- The intuition behind the median voter theorem is straightforward:
  - If we can order each individual's preferences along a line, there will be (K-1)/2 individuals who prefer the median voter's choice to each of the other (K-1)/2 individuals' choices
  - Therefore, any alternative that is not the choice of the median voter will not receive the support of at least (K-1)/2 voters to the right or left of the median voter, plus the support of the median voter.


Preferred tax rate
Median voter
0
1
Distribution of voters



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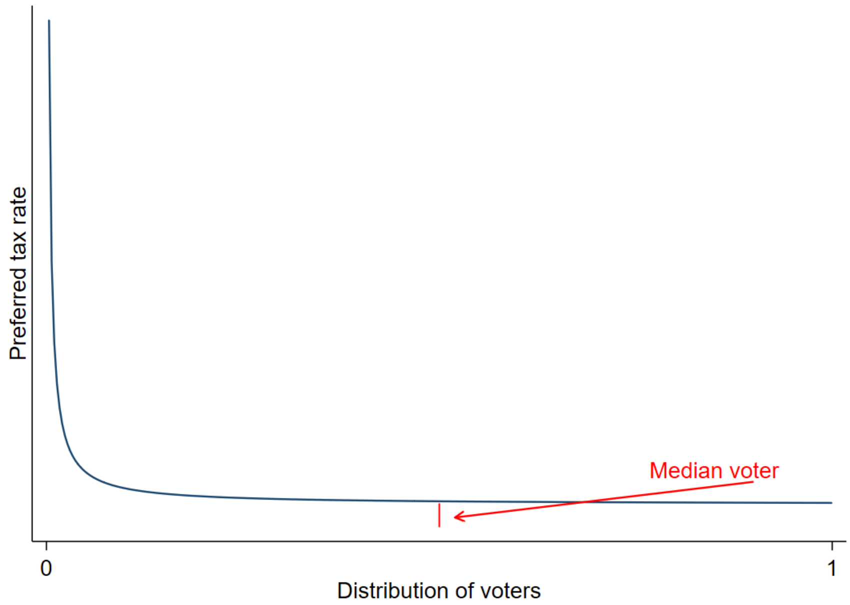

Preferred tax rate
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Distribution of voters